Goodness-of-fit tests to location-scale families based on OWA functions

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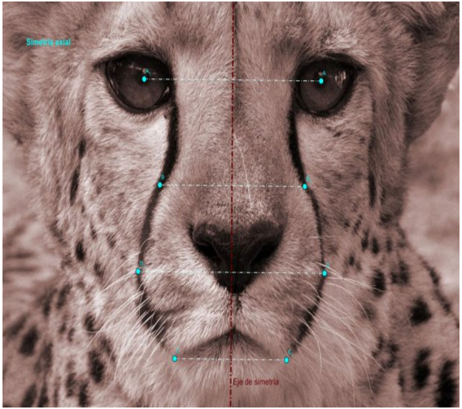
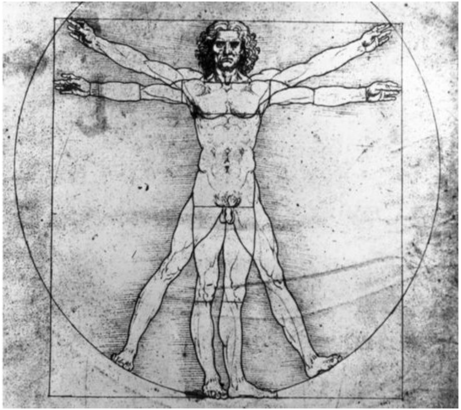
Symmetry in nature



Symmetry in art

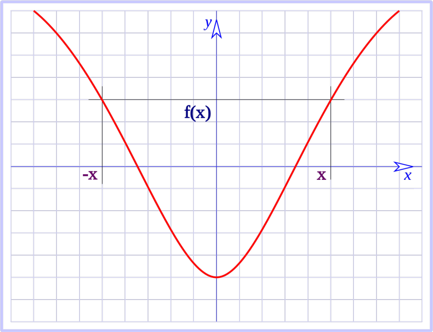


Vitruvian man
(Leonardo Da Vinci, 1490)

The starry night
(Vincent Van Gogh, 1889)

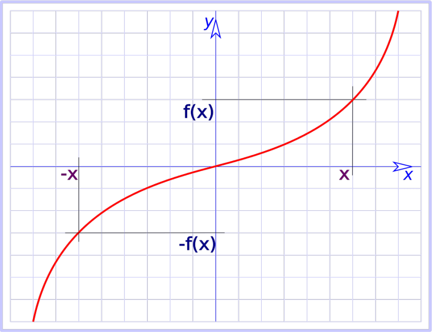
Symmetry in mathematics (analysis)

Even function



Axial symmetry about y axis

Odd function



Rotational symmetry about the origins of the coordinates

Symmetry in mathematics (linear algebra)

A matrix M is symmetric if

$$\boxed{M = M^T}$$

Typical example in probabilities: variances and covariances' matrix

$$\Sigma = \begin{pmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{XY} & \sigma_Z^2 \end{pmatrix}$$

Symmetry in a random variable

- A random variable X is called **symmetric about** $x_0 \in \mathbb{R}$ if

$$P(X \leq x_0 - x) = P(X \geq x_0 + x)\,, \text{ for any } x \in \mathbb{R}.$$

- If there exists no point $x \in \mathbb{R}$ such that $P(X = x) > 0$, then X is symmetric about $x_0 \in \mathbb{R}$ if and only if

$$F(x_0 - x) = 1 - F(x_0 + x)\,, \text{ for any } x \in \mathbb{R}.$$

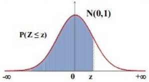
- If X is a continuous random variable with a density function f, then X is symmetric about $x_0 \in \mathbb{R}$ if and only if

$$f(x_0 - x) = f(x_0 + x)\,, \text{ for any } x \in \mathbb{R}.$$

A random variable X is called **symmetric** if there exists $x_0 \in \mathbb{R}$ about which X is symmetric.

Tables for symmetric variables: half the effort!

FUNCIÓN DE DISTRIBUCIÓN NORMAL N(0,1)



z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,99653	0,99664	0,99674	0,99683	0,99693	0,99702	0,99711	0,99720	0,99728	0,99736
2,8	0,99744	0,99752	0,99760	0,99767	0,99774	0,99781	0,99788	0,99795	0,99801	0,99807
2,9	0,99813	0,99819	0,99825	0,99831	0,99836	0,99841	0,99846	0,99851	0,99856	0,99861
3,0	0,99865	0,99869	0,99874	0,99878	0,99882	0,99886	0,99889	0,99893	0,99896	0,99900
3,1	0,99903	0,99906	0,99910	0,99913	0,99916	0,99918	0,99921	0,99924	0,99926	0,99929
3,2	0,99931	0,99934	0,99936	0,99938	0,99940	0,99942	0,99944	0,99946	0,99948	0,99950
3,3	0,99952	0,99953	0,99955	0,99957	0,99958	0,99960	0,99961	0,99962	0,99964	0,99965
3,4	0,99966	0,99968	0,99969	0,99970	0,99971	0,99972	0,99973	0,99974	0,99975	0,99976
3,5	0,99977	0,99978	0,99978	0,99979	0,99980	0,99981	0,99981	0,99982	0,99983	0,99983
3,6	0,99984	0,99985	0,99985	0,99986	0,99986	0,99987	0,99987	0,99988	0,99988	0,99989
3,7	0,99989	0,99990	0,99990	0,99990	0,99991	0,99991	0,99992	0,99992	0,99992	0,99992
3,8	0,99993	0,99993	0,99993	0,99994	0,99994	0,99994	0,99994	0,99995	0,99995	0,99995
3,9	0,99995	0,99995	0,99996	0,99996	0,99996	0,99996	0,99996	0,99996	0,99997	0,99997
4,0	0,99997	0,99997	0,99997	0,99997	0,99997	0,99997	0,99998	0,99998	0,99998	0,99998

Necessary assumption for some non-parametric tests

Wilcoxon signed-rank test (Wilcoxon, 1945): Let X be a continuous and **symmetric** random variable about which we test

$$\begin{aligned} H_0 &: \quad \mathrm{Me} = m \\ H_1 &: \quad \mathrm{Me} \neq m \end{aligned}$$

Using the statistic

$$T^+ = \sum_{X_i > m} \mathrm{rank}(|X_i - m|)\,,$$

and the

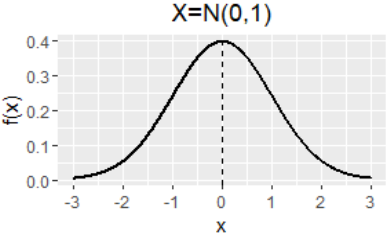
$$RR = \{T^+ < c_1, T^+ > c_2\}\,.$$

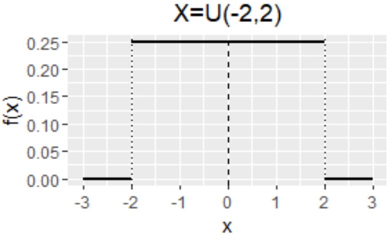
Fun fact: If we knew the population median instead of the symmetry property, we may perform a test of symmetry (Thas, Rayner and Best, 2015).

An axiomatization of the notion of symmetry (Oja, 1981)

A skewness coefficient is a function $\gamma : \mathcal{L}(\Omega, \mathcal{A}, P) \longrightarrow \mathbb{R}$

1. $\gamma(X) = 0$ if X is symmetric.





2. $\gamma(cX + d) = \gamma(X)$ for any $c, d \in \mathbb{R}$ such that $c > 0$.
3. $\gamma(-X) = -\gamma(X)$
4. $\gamma(X) \leq \gamma(Y)$ if $X \precsim Y$ with $\precsim$ an order convexity.

Sometimes, it is requiered

5. $\gamma(X) \in [-1, 1]$.

EUSFLAT 2025

Marina Iturrate–Bobes

Motivation

Skewness coefficients based on OWA functions

New family of coefficients

Sample version

Goodness-of-fit tests to location-scale families

Normal distribution

Uniform distribution

Exponential distribution

Conclusions

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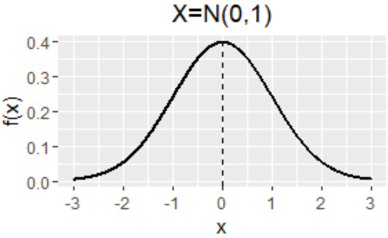

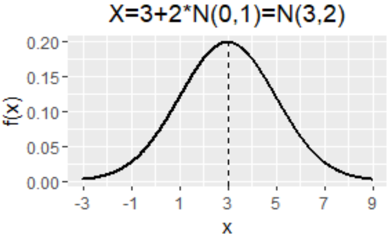

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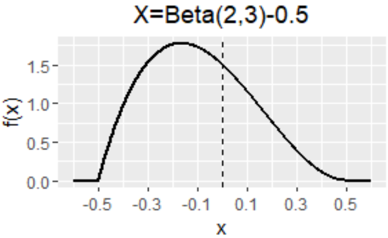

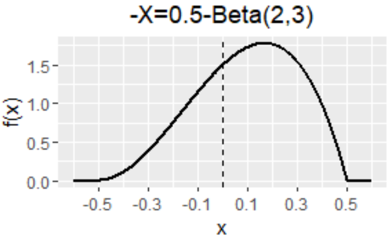

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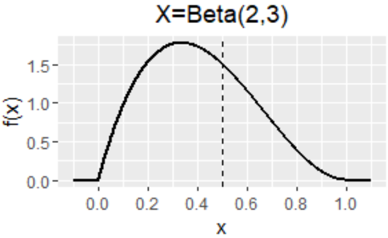

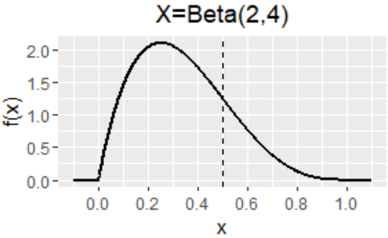

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Some classic skewness coefficients

- **Moment skewness coefficient** (Pearson, 1895)

$$\gamma_{\mathrm{M}}(X) = \frac{E\left((X-\mu)^3\right)}{\sigma^3}$$

- **Non-parametric skewness coefficient** (Pearson, 1895)

$$\gamma_{\mathrm{NP}}(X) = \frac{\mu-\mathrm{Me}(X)}{\sigma}$$

- **Groeneveld-Meeden skewness coefficient** (1984, 1995)

$$\gamma_{\mathrm{GM}}(X) = \frac{\mu-\mathrm{Me}(X)}{E(|X-\mathrm{Me}(X)|)}$$

Some robust skewness coefficients

Hinkley skewness coefficients (1975)

For any $p \in]0,1[$

$$\gamma_p(X) = \frac{(C_{1-p}(X) - C_{0.5}(X)) - (C_{0.5}(X) - C_p(X))}{C_{1-p}(X) - C_p(X)}.$$

- **Bowley skewness coefficient** (1901)

$$\gamma_{\mathrm{B}}(X) = \frac{Q_3(X)+Q_1(X)-2\mathrm{Me}(X)}{Q_3(X)-Q_1(X)}$$

- **Octile skewness coefficient** (Hinkley, 1975)

$$\gamma_{\mathrm{OCT}}(X) = \frac{\left(C_{0.875}(X)-C_{0.5}(X)\right)-\left(C_{0.5}(X)-C_{0.125}(X)\right)}{C_{0.875}(X)-C_{0.125}(X)}$$

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Weighting vectors

Basic concepts

Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^m$ be vectors:

- $\mathbf{v}$ is a weighting vector if $\mathbf{v}_i \geq 0$ for all $i \in \{1, \ldots, m\}$ and $\sum_{i=1}^{m} \mathbf{v}_i = 1$.
- $\mathbf{v}^r$ is the inverted vector of $\mathbf{v}$ if $\mathbf{v}_i^r = \mathbf{v}_{m-i+1}$ for all $i \in \{1, \ldots, m\}$.
- $\mathbf{v}$ is symmetric if $\mathbf{v} = \mathbf{v}^r$.
- $\mathbf{u}$ dominates $\mathbf{v}$ if $\sum_{i=1}^{\ell} \mathbf{v}_i \leq \sum_{i=1}^{\ell} \mathbf{u}_i$ for all $\ell \in \{1, \ldots, m\}$.

Definition of a new family of coefficients

Definition of a new family of coefficients

Let $\mathcal{L}(\Omega, \mathcal{A}, P)$ be the set of random variables over $(\Omega, \mathcal{A}, P)$:

$$\gamma(X) = \frac{(\mathbf{v} - 2\mathbf{w} + \mathbf{v}^r)^T \mathbf{C_p}(X)}{(\mathbf{v} - \mathbf{v}^r)^T \mathbf{C_p}(X)} = \frac{\mathbf{a}^T \mathbf{C_p}(X)}{\mathbf{b}^T \mathbf{C_p}(X)},$$

where

- $\mathbf{C_p}(X) \in \mathbb{R}^m$ are increasingly-ordered quantiles of X such that if $C_q(X) \in \mathbf{C_p}(X)$ for some $q \in]0,1[$, then $C_{1-q}(X) \in \mathbf{C_p}(X)$.
- $\mathbf{v} \in \mathbb{R}^m$ is an asymmetric weigthing vector such that $\sum_{i=1}^{\ell} \mathbf{v}_i \leq \sum_{i=1}^{\ell} \mathbf{v}_i^r$ for all $\ell \in \{1, \ldots, \lfloor m/2 \rfloor\}$.
- $\mathbf{w} \in \mathbb{R}^m$ is a symmetric weighting vector.

Properties of the new family of coefficients

Well-defined

The family of coefficients γ is well-defined for any absolulety continuous random variable.

Oja axioms 1, 2 and 3

The family of coefficients γ fulfills the first three axioms of Oja for any absolulety continuous random variable whose cdf is strictly increasing on the preimage of the interval $]0,1[$.

Oja axiom 4 (Theorem 1)

The family of coefficients γ with m odd and weighting vectors $\mathbf{v}$ and $\mathbf{w}$ such that $\mathbf{v}_i = 0$ for all $i \in \{1, \ldots, \lceil m/2 \rceil\}$ and $\mathbf{w} = (0, \ldots, 0, 1, 0, \ldots, 0)^T$ fulfills the four axioms of Oja for any absolulety continuous random variable whose cdf is strictly increasing on the preimage of the interval $]0,1[$ and twice differentiable.

Sample version of the new family of coefficients

Sample version

Let $\mathbf{x} \in \mathbb{R}^n$ be a simple random sample from an absolutely continuous random variable X. The sample version of γ is defined as:

$$\hat{\gamma}(\mathbf{x}) = \frac{(\mathbf{v} - 2\mathbf{w} + \mathbf{v})^T \, \widehat{\mathbf{C_p}}(\mathbf{x})}{(\mathbf{v} - \mathbf{v}^r)^T \, \widehat{\mathbf{C_p}}(\mathbf{x})} = \frac{\mathbf{a}^T \widehat{\mathbf{C_p}}(\mathbf{x})}{\mathbf{b}^T \widehat{\mathbf{C_p}}(\mathbf{x})},$$

where $\widehat{C_q}(\mathbf{x}) = \mathbf{x}_{(\lceil nq \rceil)}$ for any $q \in]0, 1[$.

A sufficient condition for it being well-defined:

$$n \geq \frac{1}{\min_{i=1,\ldots,m} \mathbf{p}_{i+1} - \mathbf{p}_i}.$$

Sample version of the new family of coefficients

Asymptotic distribution (Theorem 2)

Let X be an absolutely continuous random variable with continuous and positive density function f in a neigbourhood of $\mathbf{C}_{\mathbf{p}_i}$ for all $i \in \{1, \ldots, m\}$. Then:

$$\sqrt{n}\big(\hat{\gamma}(\mathbf{x}) - \gamma(X)\big) \xrightarrow{\mathcal{D}} \mathcal{N}(0, \tau)\,,$$

with

$$\tau^2 = \frac{4}{\left[(\mathbf{v}^T - \mathbf{v}^{r^T})\mathbf{C}_{\mathbf{p}}(X)\right]^4} \sum_{j=1}^{m} \sum_{i=1}^{m} \frac{\min\{\mathbf{p}_i, \mathbf{p}_j\}(1 - \max\{\mathbf{p}_i, \mathbf{p}_j\})}{f(\mathbf{C}_{\mathbf{p}_i}(X)) f(\mathbf{C}_{\mathbf{p}_j}(X))}$$
$$\left[\mathbf{v}_i(\mathbf{w}^T - \mathbf{v}^{r^T}) + \mathbf{v}_i^r(\mathbf{v}^T - \mathbf{w}^T) + \mathbf{w}_i(\mathbf{v}^{r^T} - \mathbf{v}^T)\right]\mathbf{C}_{\mathbf{p}}(X)$$
$$\cdot \left[\mathbf{v}_j(\mathbf{w}^T - \mathbf{v}^{r^T}) + \mathbf{v}_j^r(\mathbf{v}^T - \mathbf{w}^T) + \mathbf{w}_j(\mathbf{v}^{r^T} - \mathbf{v}^T)\right]\mathbf{C}_{\mathbf{p}}(X)\,.$$

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Goodness-of-fit tests to location-scale families

Location-scale family

Let X be a random variable. Its location-scale family is formed by those random variables for which another random variable Y exists such that $X \stackrel{\mathcal{D}}{\equiv} c + dY$ for some $c \in \mathbb{R}$ and $d \in \mathbb{R}^+$.

- Normal location-scale family.
- Uniform location-scale family.
- Exponential location-scale family.

Skewness-based goodness-of-fit tests

Let $\mathcal{F}$ be a location-scale distribution family. We can set out

$$\begin{aligned} H_0 &: \quad X \rightsquigarrow \mathcal{F} \\ H_1 &: \quad X \not\rightsquigarrow \mathcal{F} \end{aligned}$$

To perform the test, the following statistic is used

$$\sqrt{n}\frac{\hat{\gamma}(\mathbf{x}) - \gamma_{\mathcal{F}}}{\tau_{\mathcal{F}}} .$$

For any $F, F' \in \mathcal{F}$ it holds that $\gamma_F = \gamma_{F'}$ and $\tau_F = \tau_{F'}$.

Skewness-based goodness of fit tests

Asymptotic dsitribution of the test statistic

Under the conditions of Theorem 1 and given an absolutely continuos random varible X satisfying the asumptions of Theorem 2, from which a simple random sample $\mathbf{x} \in \mathbb{R}^n$ is drawn,

$$\sqrt{n}\frac{\hat{\gamma}(\mathbf{x}) - \gamma_{\mathcal{F}}}{\tau_{\mathcal{F}}} \xrightarrow[H_0]{\mathcal{D}} \mathcal{N}(0,1)\,.$$

Then, the rejection region is defined as

$$RR = \left\{ \mathbf{x} \in \mathbb{R}^n \,\middle|\, \sqrt{n}\,\frac{|\hat{\gamma}(\mathbf{x}) - \gamma_F|}{\tau_{\mathcal{F}}} > z_{1-\alpha/2} \right\}\,,$$

where $z_{1-\alpha/2}$ is the quantile of order $1-\alpha/2$ of a normal distribution.

Skewness-based goodness of fit tests

Experimental framework

- Significance level $\alpha = 0.05$.
- The power of the tests is estimated by Monte Carlo simulation (10^5 replications).
- Expected behaviour:
  1. Under H_0: Power $\in [0; 0.0511]$.
  2. Under H_1: Power ≈ 1.
- $n \in \{10, 50, 100, 200\}$.
- $\mathbf{w} = (0, 0, 1, 0, 0)^T$.
- $\mathbf{v} = \lambda\,(0, 0, 0, 1, 0)^T + (1 - \lambda)\,(0, 0, 0, 0, 1)^T$, with $\lambda \in \{0, 1/4, 1/2, 3/4, 1\}$
- $\mathbf{C_p}$ such that $\mathbf{p} = (1/6, 2/6, 3/6, 4/6, 5/6)^T$.

Goodness-of-fit to normal distribution

Distribution	n	$\mathbf{v} = (0,0,0,1,0)^T$	$\mathbf{v} = (0,0,0,3/4,1/4)^T$	$\mathbf{v} = (0,0,0,1/2,1/2)^T$	$\mathbf{v} = (0,0,0,1/4,3/4)^T$	$\mathbf{v} = (0,0,0,0,1)^T$
Normal	10	**0.0000**	**0.0219**	**0.0357**	**0.0373**	**0.0344**
	50	**0.0457**	**0.0491**	0.0514	0.0514	**0.0504**
	100	**0.0502**	**0.0494**	**0.0490**	**0.0504**	**0.0486**
	200	**0.0489**	**0.0509**	**0.0490**	**0.0497**	**0.0498**
Uniform	10	**0.0000**	**0.0325**	**0.0506**	0.0541	**0.0486**
	50	**0.0492**	0.0658	0.0702	0.0700	0.0662
	100	0.0541	0.0667	0.0694	0.0666	0.0665
	200	0.0535	0.0663	0.0713	0.0691	0.0648
Cauchy	10	**0.0000**	**0.0406**	0.0935	0.1205	0.1285
	50	**0.0498**	0.0736	0.1237	0.1566	0.1760
	100	0.0535	0.0775	0.1307	0.1685	0.1889
	200	0.0531	0.0760	0.1286	0.1681	0.1901
Logistic	10	**0.0000**	**0.0205**	**0.0346**	**0.0385**	**0.0352**
	50	**0.0434**	**0.0470**	**0.0507**	0.0531	0.0528
	100	**0.0483**	**0.0463**	**0.0490**	**0.0506**	0.0522
	200	**0.0484**	**0.0472**	**0.0501**	**0.0510**	0.0550
Exponential	10	**0.0000**	0.0997	0.1819	0.2070	0.2072
	50	0.1392	0.3457	0.5206	0.6196	0.6651
	100	0.2070	0.5604	0.7842	0.8743	0.9109
	200	0.3374	0.8045	0.9593	0.9871	0.9945

Table: Power of the goodness-of-fit test to normal distribution.

Goodness-of-fit to uniform distribution

Distribution	n	$\mathbf{v} = (0,0,0,1,0)^T$	$\mathbf{v} = (0,0,0,3/4,1/4)^T$	$\mathbf{v} = (0,0,0,1/2,1/2)^T$	$\mathbf{v} = (0,0,0,1/4,3/4)^T$	$\mathbf{v} = (0,0,0,0,1)^T$
Normal	10	**0.0000**	**0.0065**	**0.0165**	**0.0198**	**0.0200**
	50	**0.0398**	**0.0326**	**0.0338**	**0.0354**	**0.0352**
	100	**0.0459**	**0.0346**	**0.0324**	**0.0337**	**0.0349**
	200	**0.0441**	**0.0345**	**0.0347**	**0.0351**	**0.0364**
Uniform	10	**0.0000**	**0.0118**	**0.0259**	**0.0299**	**0.0296**
	50	**0.0433**	**0.0463**	**0.0507**	**0.0502**	**0.0486**
	100	**0.0498**	**0.0488**	**0.0487**	**0.0488**	**0.0479**
	200	**0.0482**	**0.0508**	**0.0499**	**0.0493**	**0.0498**
Cauchy	10	**0.0000**	**0.0169**	0.0553	0.0809	0.0930
	50	**0.0432**	0.0538	0.0939	0.1268	0.1495
	100	**0.0505**	0.0565	0.1009	0.1357	0.1608
	200	**0.0499**	0.0575	0.0990	0.1371	0.1611
Logistic	10	**0.0000**	**0.0071**	**0.0164**	**0.0199**	**0.0213**
	50	**0.0391**	**0.0316**	**0.0328**	**0.0349**	**0.0386**
	100	**0.0449**	**0.0328**	**0.0326**	**0.0333**	**0.0381**
	200	**0.0436**	**0.0341**	**0.0339**	**0.0365**	**0.0399**
Exponential	10	**0.0000**	**0.0403**	0.1037	0.1380	0.1448
	50	0.1274	0.2956	0.4561	0.5589	0.6163
	100	0.1932	0.4995	0.7338	0.8413	0.8901
	200	0.3168	0.7659	0.9428	0.9826	0.9920

Table: Power of the goodness-of-fit test to uniform distribution.

Goodness-of-fit to exponential distribution

Distribution	n	$\mathbf{v} = (0,0,0,1,0)^T$	$\mathbf{v} = (0,0,0,3/4,1/4)^T$	$\mathbf{v} = (0,0,0,1/2,1/2)^T$	$\mathbf{v} = (0,0,0,1/4,3/4)^T$	$\mathbf{v} = (0,0,0,0,1)^T$
Normal	10	**0.0032**	**0.0183**	**0.0444**	0.0757	0.1037
	50	0.0685	0.2043	0.3730	0.5002	0.5868
	100	0.1310	0.4225	0.6866	0.8249	0.8860
	200	0.2569	0.7438	0.9464	0.9857	0.9948
Uniform	10	**0.0036**	**0.0230**	0.0526	0.0833	0.1118
	50	0.0694	0.2184	0.3760	0.4921	0.5718
	100	0.1378	0.4243	0.6708	0.8019	0.8652
	200	0.2578	0.7254	0.9285	0.9795	0.9916
Cauchy	10	**0.0042**	0.0514	0.1249	0.1793	0.2182
	50	0.0742	0.2449	0.4167	0.5142	0.5693
	100	0.1395	0.4447	0.6560	0.7494	0.7926
	200	0.2627	0.7274	0.8963	0.9432	0.9587
Logistic	10	**0.0036**	**0.0176**	**0.0467**	0.0812	0.1100
	50	0.0683	0.2045	0.3762	0.5037	0.5844
	100	0.1315	0.4236	0.6901	0.8245	0.8812
	200	0.2563	0.7463	0.9463	0.9865	0.9945
Exponential	10	**0.0021**	**0.0048**	**0.0086**	**0.0127**	**0.0166**
	50	**0.0363**	**0.0399**	**0.0406**	**0.0424**	**0.0425**
	100	**0.0468**	**0.0445**	**0.0452**	**0.0439**	**0.0447**
	200	**0.0474**	**0.0471**	**0.0483**	**0.0476**	**0.0484**

Table: Power of the goodness-of-fit test to exponential distribution.

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Conclusions

Conclusions of the experimental setup

- Significance level is preserved under H_0 for large enough sample sizes.
- Location-scale families in which the skewness coefficient takes the same value cannot be distinguished.
- In the case of symmetric distributions, the power is lower than that of the family under H_0 if $\tau^2_{\mathcal{F}}$ is smaller than that of the family under H_0.

Future work

- To generalise the definition of the skewness coefficient to a **wider range of weighting vectors**.
- To explore the use of different skewness coefficients constructed from **different aggregation functions**.
- To combine the statistic with a **kurtosis coefficient** (Jarque-Bera test) for performing goodness-of-fit tests to location-scale families.
- To apply the skewness coefficient in **symmetry testing** procedures.

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