Asymptotic confidence intervals for a skewness parameter by using OWA functions

Marina Iturrate–Bobes, Ignacio Montes and Raúl Pérez–Fernández

University of Oviedo

Universidad de Oviedo

21st International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems
Rome, Italy
June 15th–19th 2026

Index

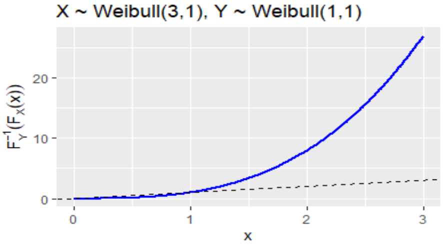
Index

The convex order of van Zwet (1964)

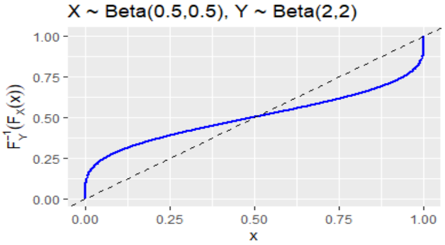
Definition

Let $\mathcal{L}'(\Omega, \mathcal{A}, P)$ be the set of random variables over $(\Omega, \mathcal{A}, P)$ for which the cdf is absolutely continuous, strictly increasing on the preimage of $]0,1[$ and twice differentiable. For any $X, Y \in \mathcal{L}'(\Omega, \mathcal{A}, P)$, we say that X is not more strongly skew to the right than Y, denoted by $X \precsim Y$, if $F_Y^{-1} \circ F_X$ is a convex function on the support of X.

$X \precsim Y$



$X \not\precsim Y$

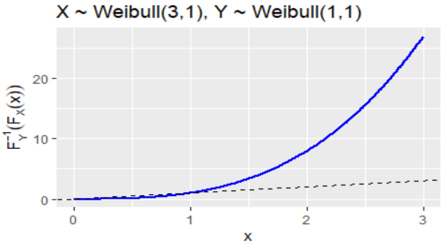


Equivalent definition of the order in terms of concavity

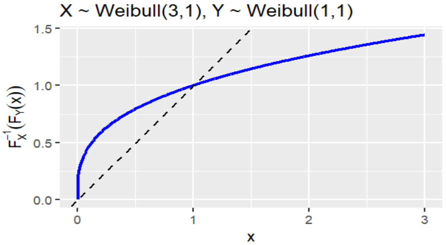
Alternative definition

$F_Y^{-1} \circ F_X$ is a convex function on the support of X if and only if $F_X^{-1} \circ F_Y$ is a concave function on the support of Y. Therefore, saying that $F_Y^{-1} \circ F_X$ is a convex or concave function on the support of X means that $X \precsim Y$ or $Y \precsim X$.

$F_Y^{-1} \circ F_X$ convex



$F_X^{-1} \circ F_Y$ concave



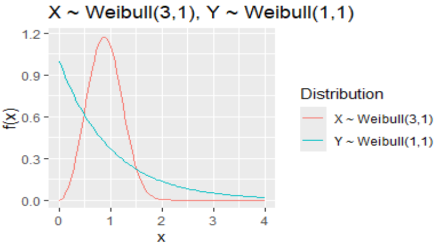
Interpretation of the convex order of van Zwet

Interpretation

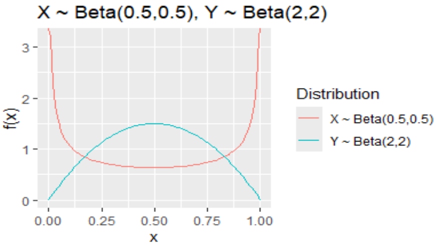
X being not more strongly skew to the right than Y intuitively means that Y has a longer tail of extreme values at the right end compared to X.

Skew has no relation with location or scale (it is invariant to translations and scalings).

$X \precsim Y$



$X \not\precsim Y$

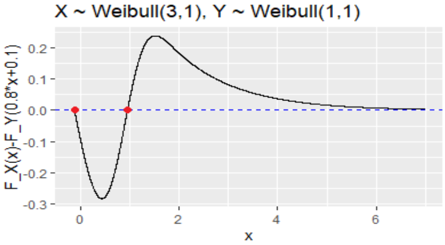


Characterization of the order of van Zwet

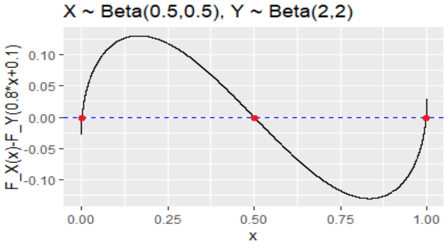
Characterization

An equivalent definition of $X \precsim Y$ is that, for any $a, b \in \mathbb{R}$ with $a > 0$, $F_X(x)$ and $F_Y(ax+b)$ cross each other at most twice with the sequence of changes of sign of $F_X(x) - F_Y(ax+b)$ (ignoring zeros) being positive to negative to positive.

$X \precsim Y$



$X \not\precsim Y$

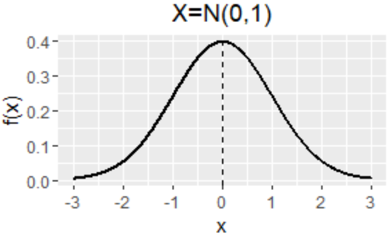


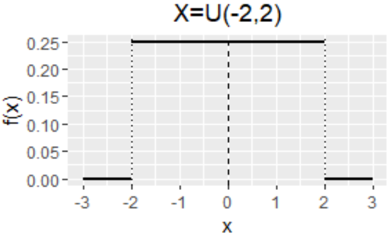
An axiomatization of the notion of skewness (Oja, 1981)

Let $\mathcal{L}(\Omega, \mathcal{A}, P)$ be the set of random variables over $(\Omega, \mathcal{A}, P)$.

A skewness coefficient is a function $\gamma : \ \mathcal{L}(\Omega, \mathcal{A}, P) \longrightarrow \mathbb{R}$

1. $\gamma(X) = 0$ if X is symmetric.





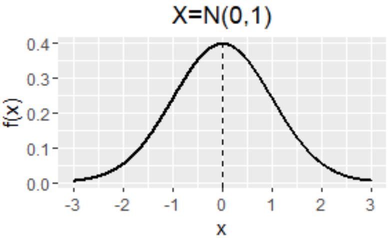
2. $\gamma(cX + d) = \gamma(X)$ for any $c, d \in \mathbb{R}$ such that $c > 0$.
3. $\gamma(-X) = -\gamma(X)$
4. $\gamma(X) \leq \gamma(Y)$ if $X \precsim Y$.

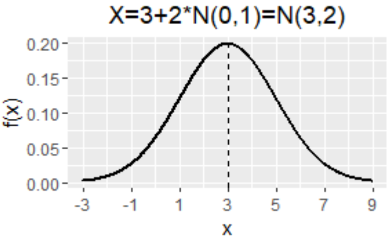
An axiomatization of the notion of skewness (Oja, 1981)

Let $\mathcal{L}(\Omega, \mathcal{A}, P)$ be the set of random variables over $(\Omega, \mathcal{A}, P)$.

A skewness coefficient is a function $\gamma: \ \mathcal{L}(\Omega, \mathcal{A}, P) \longrightarrow \mathbb{R}$

1. $\gamma(X) = 0$ if X is symmetric.
2. $\gamma(cX + d) = \gamma(X)$ for any $c, d \in \mathbb{R}$ such that $c > 0$.





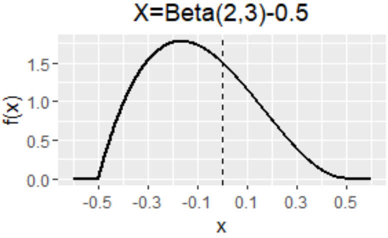
3. $\gamma(-X) = -\gamma(X)$.
4. $\gamma(X) \leq \gamma(Y)$ if $X \precsim Y$.

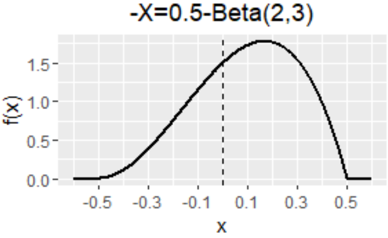
An axiomatization of the notion of skewness (Oja, 1981)

Let $\mathcal{L}(\Omega, \mathcal{A}, P)$ be the set of random variables over $(\Omega, \mathcal{A}, P)$.

A skewness coefficient is a function $\gamma : \mathcal{L}(\Omega, \mathcal{A}, P) \longrightarrow \mathbb{R}$

1. $\gamma(X) = 0$ if X is symmetric.
2. $\gamma(cX + d) = \gamma(X)$ for any $c, d \in \mathbb{R}$ such that $c > 0$.
3. $\gamma(-X) = -\gamma(X)$.





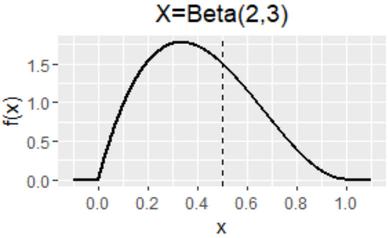
4. $\gamma(X) \leq \gamma(Y)$ if $X \precsim Y$.

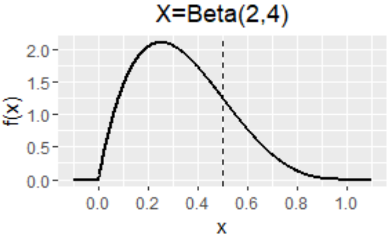
An axiomatization of the notion of skewness (Oja, 1981)

Let $\mathcal{L}(\Omega, \mathcal{A}, P)$ be the set of random variables over $(\Omega, \mathcal{A}, P)$.

A skewness coefficient is a function $\gamma : \mathcal{L}(\Omega, \mathcal{A}, P) \longrightarrow \mathbb{R}$

1. $\gamma(X) = 0$ if X is symmetric.
2. $\gamma(cX + d) = \gamma(X)$ for any $c, d \in \mathbb{R}$ such that $c > 0$.
3. $\gamma(-X) = -\gamma(X)$.
4. $\gamma(X) \leq \gamma(Y)$ if $X \precsim Y$.





Location-scale-skewness families

Definition

A location-scale-skewness family is a family $\mathcal{F} \subseteq \mathcal{L}'(\Omega, \mathcal{A}, P)$ for which:

- For any $X \in \mathcal{F}$, $c \in \mathbb{R}^+$ and $d \in \mathbb{R}$, it holds that $cX + d \in \mathcal{F}$.
- For any $X, Y \in \mathcal{F}$, it either holds that $F_Y^{-1} \circ F_X$ is a convex or concave function on the support of X.

Examples

In the Weibull family all distribution are comparable in terms of skewness, therefore the shifted-Weibull distribution forms a location-scale-skewness family.

On the contrary, it has been shown that there are parameters of the beta distribution that lead to incomparability in terms of skewness, therefore the beta distribution does not form a location-scale-skewness family.

Types of location-scale-skewness families

Location-scale-skewness families of interest

We devote our attention to location-scale-skewness families parametrized by a location parameter μ, a scale parameter σ and a skewness parameter δ so that:

Type A: $F_Y^{-1} \circ F_X$ is convex for any $X \sim \mathcal{F}_{\mu,\sigma,\delta}$ and $Y \sim \mathcal{F}_{\mu',\sigma',\delta'}$ with $\delta \leq \delta'$.

In location-scale-skewness families of Type A, the skewness is constant with respect to the location and scale and increases with δ, i.e., skewness can be seen as an increasing function $\phi(\delta) := \gamma(X)$.

Type B: $F_Y^{-1} \circ F_X$ is concave for any $X \sim \mathcal{F}_{\mu,\sigma,\delta}$ and $Y \sim \mathcal{F}_{\mu',\sigma',\delta'}$ with $\delta \leq \delta'$.

In location-scale-skewness families of Type B, the skewness is constant with respect to the location and scale and decreases with δ, i.e., skewness can be seen as a decreasing function $\varphi(\delta) := \gamma(X)$.

Index

A new family of coefficients

Definition of a new family of coefficients (Iturrate-Bobes et al., 2025)

For $X \in \mathcal{L}(\Omega, \mathcal{A}, P)$, we define the family of coefficients:

$$\gamma(X) = \frac{\left((\mathbf{v} - \mathbf{w}) - (\mathbf{w} - \mathbf{v}^r)\right)^T \mathbf{C_p}(X)}{(\mathbf{v} - \mathbf{v}^r)^T \mathbf{C_p}(X)},$$

where

- $\mathbf{C_p}(X) \in \mathbb{R}^m$ are quantiles of X arranged in ascending order such that if $C_q(X) \in \mathbf{C_p}(X)$ for some $q \in]0,1[$, then $C_{1-q}(X) \in \mathbf{C_p}(X)$.
- $\mathbf{v} \in \mathbb{R}^m$ is an asymmetric weigthing vector such that $\sum_{i=1}^{\ell} \mathbf{v}_i \leq \sum_{i=1}^{\ell} \mathbf{v}_i^r$ for all $\ell \in \{1, \ldots, \lfloor m/2 \rfloor\}$.
- $\mathbf{w} \in \mathbb{R}^m$ is a symmetric weighting vector.

Properties of the new family of coefficients

Well-defined

The family of coefficients γ is well-defined for any absolulety continuous random variable.

Oja axioms 1, 2 and 3

The family of coefficients γ fulfills the first three axioms of Oja for any absolulety continuous random variable whose cdf is strictly increasing on the preimage of the interval $]0, 1[$.

Oja axiom 4

The family of coefficients γ with m odd and weighting vectors $\mathbf{v}$ and $\mathbf{w}$ such that $\mathbf{v}_i = 0$ for all $i \in \{1, \ldots, \lceil m/2 \rceil\}$ and $\mathbf{w} = (0, \ldots, 0, 1, 0, \ldots, 0)^T$ fulfills the four axioms of Oja for any absolulety continuous random variable whose cdf is strictly increasing on the preimage of the interval $]0, 1[$ and twice differentiable (i.e., if we are in $\mathcal{L}'(\Omega, \mathcal{A}, P)$).

The sample version is based on OWAs

Definition

Let $\mathbf{X} \in \mathbb{R}^n$ be a random sample from $X \in \mathcal{L}(\Omega, \mathcal{A}, P)$. The sample version of γ is defined as:

$$\hat{\gamma}(\mathbf{X}) = \frac{\left((\mathbf{v} - \mathbf{w}) - (\mathbf{w} - \mathbf{v}^r)\right)^T \mathbf{X}_{(\lceil n\mathbf{p} \rceil)}}{(\mathbf{v} - \mathbf{v}^r)^T \mathbf{X}_{(\lceil n\mathbf{p} \rceil)}} .$$

A sufficient condition for it to be well-defined (almost surely):

$$X \in \mathcal{L}'(\Omega, \mathcal{A}, P) \qquad \text{and} \qquad n \geq \frac{1}{\min_{i=1,\ldots,m} \mathbf{p}_{i+1} - \mathbf{p}_i} .$$

Asymptotic distribution

Proposition

Let $\mathbf{X} \in \mathbb{R}^n$ be a random sample from $X \in \mathcal{L}'(\Omega, \mathcal{A}, P)$. It holds that

$$\sqrt{n}\big(\hat{\gamma}(\mathbf{X}) - \gamma(X)\big) \xrightarrow{\mathcal{D}} \mathcal{N}(0, \tau)\,,$$

with

$$\tau^2 = \frac{4}{\left[(\mathbf{v}^T - \mathbf{v}^{r^T})\mathbf{C}_{\mathbf{p}}(X)\right]^4} \sum_{j=1}^{m} \sum_{i=1}^{m} \frac{\min\{\mathbf{p}_i, \mathbf{p}_j\}(1 - \max\{\mathbf{p}_i, \mathbf{p}_j\})}{f(\mathbf{C}_{\mathbf{p}_i}(X)) f(\mathbf{C}_{\mathbf{p}_j}(X))}$$
$$\cdot \left[\mathbf{v}_i(\mathbf{w}^T - \mathbf{v}^{r^T}) + \mathbf{v}_i^r(\mathbf{v}^T - \mathbf{w}^T) + \mathbf{w}_i(\mathbf{v}^{r^T} - \mathbf{v}^T)\right]\mathbf{C}_{\mathbf{p}}(X)$$
$$\cdot \left[\mathbf{v}_j(\mathbf{w}^T - \mathbf{v}^{r^T}) + \mathbf{v}_j^r(\mathbf{v}^T - \mathbf{w}^T) + \mathbf{w}_j(\mathbf{v}^{r^T} - \mathbf{v}^T)\right]\mathbf{C}_{\mathbf{p}}(X)\,.$$

PROBLEM: τ is unknown in practice.

Estimating the asymptotic variance

Theorem

Let $\mathbf{X} \in \mathbb{R}^n$ be a random sample from $X \in \mathcal{L}'(\Omega, \mathcal{A}, P)$. It holds that

$$\sqrt{n}\frac{\hat{\gamma}(\mathbf{X}) - \gamma(X)}{\hat{\tau}(\mathbf{X})} \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1)\,,$$

with

$$\begin{aligned}\hat{\tau}^2(\mathbf{X}) = & \frac{4}{\left[(\mathbf{v}^T - \mathbf{v}^{r^T})X_{(\lceil n\mathbf{p}\rceil)}\right]^4} \sum_{j=1}^{m}\sum_{i=1}^{m} \frac{\min\{\mathbf{p}_i, \mathbf{p}_j\}(1 - \max\{\mathbf{p}_i, \mathbf{p}_j\})}{f_n(\mathbf{X}_{(\lceil n\mathbf{p}\rceil)})f_n(\mathbf{X}_{(\lceil n\mathbf{p}\rceil)})} \\ & \cdot \left[\mathbf{v}_i(\mathbf{w}^T - \mathbf{v}^{r^T}) + \mathbf{v}_i^r(\mathbf{v}^T - \mathbf{w}^T) + \mathbf{w}_i(\mathbf{v}^{r^T} - \mathbf{v}^T)\right]\mathbf{X}_{(\lceil n\mathbf{p}\rceil)} \\ & \cdot \left[\mathbf{v}_j(\mathbf{w}^T - \mathbf{v}^{r^T}) + \mathbf{v}_j^r(\mathbf{v}^T - \mathbf{w}^T) + \mathbf{w}_j(\mathbf{v}^{r^T} - \mathbf{v}^T)\right]\mathbf{X}_{(\lceil n\mathbf{p}\rceil)}\,,\end{aligned}$$

where f_n is a kernel-based density estimate of the probability density function of X.

Confidence intervals for the skewness coefficient

Proposition

Given a random sample $\mathbf{X}^T = (\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n)$ from a random variable $X \in \mathcal{L}'(\Omega, \mathcal{A}, P)$, an asymptotic confidence interval for $\gamma(X)$ with confidence level $1 - \alpha$ is:

$$\left[\hat{\gamma}(\mathbf{X}) - \frac{\hat{\tau}(\mathbf{X})\, z_{1-\alpha_2}}{\sqrt{n}}\,,\ \hat{\gamma}(\mathbf{X}) - \frac{\hat{\tau}(\mathbf{X})\, z_{\alpha_1}}{\sqrt{n}}\right],$$

where $\alpha_1, \alpha_2 \in [0, \alpha]$ are such that $\alpha_1 + \alpha_2 = \alpha$, z_p is the quantile of order p of the standard normal distribution, and $\hat{\tau}^2(\mathbf{X})$ as defined before.

Remark

It has been said that for location-scale-skewness families of Type A or Type B, $\gamma(X)$ is a monotone function of δ. The lower and upper generalized inverses of a monotone function may be used for further extending the result.

Confidence intervals for the skewness parameter

Theorem

Consider a location-scale-skewness family $\mathcal{F} \subseteq \mathcal{L}'(\Omega, \mathcal{A}, P)$ of Type A or Type B. Given a random sample $\mathbf{X}^T = (\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n)$ from a random variable $X \in \mathcal{F}$, an asymptotic confidence interval for δ with confidence level $1 - \alpha$ is:

$$\text{Type A: } \left[\phi_{\downarrow}^{-1}\left(\hat{\gamma}(\mathbf{X}) - \frac{\hat{\tau}(\mathbf{X})\, z_{1-\alpha_2}}{\sqrt{n}}\right),\ \phi_{\uparrow}^{-1}\left(\hat{\gamma}(\mathbf{X}) - \frac{\hat{\tau}(\mathbf{X})\, z_{\alpha_1}}{\sqrt{n}}\right)\right],$$

$$\text{Type B: } \left[\varphi_{\downarrow}^{-1}\left(\hat{\gamma}(\mathbf{X}) - \frac{\hat{\tau}(\mathbf{X})\, z_{\alpha_1}}{\sqrt{n}}\right),\ \varphi_{\uparrow}^{-1}\left(\hat{\gamma}(\mathbf{X}) - \frac{\hat{\tau}(\mathbf{X})\, z_{1-\alpha_2}}{\sqrt{n}}\right)\right],$$

where the endpoints of the confidence interval are taken to be open whenever they are infinite, $\alpha_1, \alpha_2 \in [0, \alpha]$ are such that $\alpha_1 + \alpha_2 = \alpha$, z_p is the quantile of order p of the standard normal distribution, and $\hat{\tau}^2(\mathbf{X})$ as defined before.

Index

Case study

The Weibull distribution as a location-scale-skewness family

The Weibull distribution is an example of location-scale-skewness family of Type B. Therefore, it follows that

$$\varphi(\delta) = \gamma(X) = \frac{(\mathbf{v} - 2\mathbf{w} + \mathbf{v}^{\mathrm{r}})^T\big(-\ln(\mathbf{1} - \mathbf{p})\big)^{1/\delta}}{(\mathbf{v} - \mathbf{v}^{\mathrm{r}})^T\big(-\ln(\mathbf{1} - \mathbf{p})\big)^{1/\delta}}$$

is decreasing. Actually, φ is strictly decreasing, and, thus, the lower and upper generalized upper inverses $\varphi_{\downarrow}^{-1}$ and $\varphi_{\uparrow}^{-1}$ of φ coincide with the inverse φ^{-1} of φ.

Experiments

Set-up

- Confidence level $1 - \alpha = 0.95$.
- The coverage is estimated through Monte Carlo simulation with 10^5 replications.
- $\delta \in \{0.5, 1, 1.5, \ldots, 6\}$.
- $n \in \{25, 50, 100, 200\}$.
- $\mathbf{C_p}$ is such that $\mathbf{p} = (1/6, 2/6, 3/6, 4/6, 5/6)^T$.
- $\mathbf{w} = (0, 0, 1, 0, 0)^T$.
- $\mathbf{v} = \lambda\,(0, 0, 0, 1, 0)^T + (1 - \lambda)\,(0, 0, 0, 0, 1)^T$, with $\lambda \in \{0, 1/4, 1/2, 3/4, 1\}$.
- All choices of kernel and bandwidth selector available in the R function *density()* were explored.

Case (i) Empirical coverage for $\alpha_1 = \alpha_2 = {}^{\alpha}/_{2}$

		δ											
λ	n	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
0	25	0.94784	0.94905	0.95191	0.95356	0.95628	0.95579	0.95874	0.95845	0.96028	0.95992	0.96224	0.96240
	50	0.93511	0.93738	0.94617	0.94863	0.94919	0.95085	0.95109	0.95154	0.95313	0.95339	0.95387	0.95527
	100	0.92069	0.93802	0.94628	0.94971	0.95030	0.95160	0.95077	0.95288	0.95269	0.95261	0.95420	0.95483
	200	0.92017	0.94122	0.94971	0.95166	0.95294	0.95214	0.95473	0.95363	0.95371	0.95363	0.95491	0.95475
$^1/_4$	25	0.94468	0.95311	0.95516	0.95675	0.95684	0.95910	0.95912	0.96085	0.96204	0.96186	0.96337	0.96391
	50	0.94458	0.94534	0.94784	0.95012	0.95101	0.94983	0.95207	0.95165	0.95057	0.95312	0.95312	0.95352
	100	0.92791	0.94417	0.95077	0.95219	0.95241	0.95356	0.95357	0.95418	0.95348	0.95456	0.95455	0.95476
	200	0.90451	0.94391	0.95151	0.95417	0.95301	0.95287	0.95286	0.95424	0.95339	0.95304	0.95395	0.95468
$^1/_2$	25	0.94301	0.95480	0.95779	0.95816	0.95809	0.96006	0.96065	0.96200	0.96391	0.96357	0.96401	0.96476
	50	0.94263	0.94732	0.94930	0.95031	0.95080	0.94991	0.95220	0.95108	0.95205	0.95234	0.95243	0.95365
	100	0.92263	0.94525	0.95188	0.95364	0.95364	0.95319	0.95370	0.95346	0.95347	0.95421	0.95421	0.95377
	200	0.87378	0.93789	0.95065	0.95352	0.95402	0.95382	0.95379	0.95269	0.95295	0.95318	0.95176	0.95213
$^3/_4$	25	0.89915	0.95670	0.96657	0.97014	0.97173	0.97264	0.97312	0.97378	0.97411	0.97394	0.97505	0.97572
	50	0.86912	0.94780	0.95794	0.95933	0.95879	0.95666	0.95564	0.95490	0.95429	0.95343	0.95437	0.95206
	100	0.71962	0.91265	0.95087	0.95833	0.95866	0.95923	0.95679	0.95514	0.95511	0.95415	0.95225	0.95112
	200	0.40843	0.81688	0.92450	0.95024	0.95569	0.95686	0.95368	0.95136	0.95102	0.94785	0.94553	0.94408
1	25	0.84235	0.95324	0.97489	0.97999	0.98233	0.98349	0.98333	0.98461	0.98400	0.98448	0.98493	0.98484
	50	0.74577	0.92888	0.96128	0.96700	0.96624	0.96556	0.96386	0.96216	0.96025	0.95897	0.95653	0.95760
	100	0.44053	0.84885	0.94113	0.95931	0.96306	0.96307	0.96184	0.95987	0.95611	0.95480	0.95116	0.95128
	200	0.10767	0.66755	0.88962	0.94275	0.95536	0.95775	0.95619	0.95253	0.94876	0.94444	0.94232	0.93756

Case (i) Estimated proportion of unbounded confidence intervals for $\alpha_1 = \alpha_2 = \alpha/2$

		δ											
λ	n	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
0	25	0.07035	0.44289	0.69915	0.81197	0.86821	0.89765	0.91911	0.92908	0.93847	0.94216	0.94864	0.95132
	50	0.00118	0.11282	0.38110	0.58678	0.71246	0.78468	0.82971	0.85984	0.87867	0.89226	0.90502	0.91321
	100	0.00000	0.00713	0.11635	0.32431	0.50497	0.63165	0.71810	0.77741	0.81403	0.84240	0.86347	0.88023
	200	0.00000	0.00001	0.00867	0.08275	0.23448	0.39063	0.51751	0.61837	0.68835	0.73896	0.77913	0.80915
$1/4$	25	0.08570	0.47333	0.71431	0.82070	0.87630	0.90328	0.91983	0.93024	0.93928	0.94356	0.94839	0.95167
	50	0.00260	0.13439	0.39906	0.59472	0.71215	0.77966	0.82268	0.85112	0.86988	0.88677	0.89604	0.90539
	100	0.00001	0.01217	0.14168	0.34752	0.51882	0.63598	0.71583	0.76895	0.80685	0.83476	0.85444	0.87210
	200	0.00000	0.00006	0.01409	0.10061	0.24522	0.39201	0.51131	0.60314	0.67080	0.72250	0.75889	0.78925
$1/2$	25	0.09474	0.49024	0.72588	0.82829	0.87789	0.90558	0.92111	0.93233	0.93792	0.94363	0.94884	0.95188
	50	0.00301	0.14435	0.41500	0.60392	0.71684	0.78140	0.82049	0.84804	0.87070	0.88352	0.89288	0.90359
	100	0.00000	0.01591	0.15562	0.36452	0.52868	0.64282	0.71872	0.76947	0.80456	0.83277	0.85053	0.86762
	200	0.00000	0.00011	0.01697	0.11137	0.25614	0.40255	0.51760	0.60813	0.66905	0.71988	0.75305	0.78452
$3/4$	25	0.23641	0.66661	0.82496	0.88801	0.91716	0.93264	0.94131	0.94860	0.95311	0.95766	0.95975	0.96202
	50	0.02157	0.30907	0.56827	0.70138	0.77506	0.81547	0.84121	0.86145	0.87526	0.88405	0.89350	0.89889
	100	0.00062	0.10414	0.35453	0.53856	0.65233	0.71945	0.76858	0.79879	0.82148	0.83831	0.85245	0.86199
	200	0.00000	0.00801	0.11268	0.27442	0.42162	0.52811	0.60341	0.65987	0.69861	0.73016	0.75426	0.77389
1	25	0.64001	0.84238	0.91230	0.93901	0.95311	0.95975	0.96371	0.96776	0.96996	0.97206	0.97360	0.97463
	50	0.23491	0.58834	0.74680	0.81331	0.84897	0.86895	0.88491	0.89226	0.89986	0.90498	0.90933	0.91340
	100	0.07178	0.42881	0.63916	0.73546	0.78552	0.81527	0.83683	0.85174	0.85974	0.86801	0.87296	0.88116
	200	0.00372	0.18126	0.41255	0.55717	0.63503	0.68752	0.72280	0.74821	0.76733	0.78371	0.79566	0.80094

Case (ii) Empirical coverage for $\alpha_1 = 0$ and $\alpha_2 = \alpha$

		δ											
λ	n	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
0	25	0.95643	0.94887	0.95320	0.95308	0.95310	0.95521	0.95480	0.95459	0.95474	0.95506	0.95534	0.95571
	50	0.92350	0.91925	0.92846	0.93213	0.93583	0.93506	0.93710	0.93904	0.93751	0.93889	0.93900	0.93932
	100	0.90276	0.91844	0.93202	0.93508	0.93971	0.94212	0.94200	0.94305	0.94324	0.94466	0.94567	0.94462
	200	0.91371	0.92850	0.93908	0.94253	0.94470	0.94519	0.94559	0.94527	0.94618	0.94626	0.94760	0.94743
$1/4$	25	0.95627	0.95020	0.95239	0.95339	0.95286	0.95523	0.95376	0.95548	0.95513	0.95492	0.95685	0.95377
	50	0.92265	0.91903	0.92863	0.93350	0.93509	0.93704	0.93860	0.93741	0.93805	0.93861	0.93883	0.93831
	100	0.90309	0.91863	0.93131	0.93522	0.93943	0.94035	0.94274	0.94241	0.94381	0.94328	0.94460	0.94409
	200	0.91317	0.92845	0.93771	0.94242	0.94352	0.94454	0.94524	0.94632	0.94637	0.94851	0.94656	0.94806
$1/2$	25	0.95771	0.95085	0.95186	0.95377	0.95407	0.95442	0.95425	0.95492	0.95595	0.95546	0.95599	0.95435
	50	0.92387	0.92064	0.92953	0.93256	0.93417	0.93614	0.93635	0.93866	0.93905	0.93904	0.93943	0.94005
	100	0.90197	0.91628	0.92967	0.93556	0.93909	0.93970	0.94194	0.94296	0.94322	0.94529	0.94438	0.94487
	200	0.91447	0.92788	0.93881	0.94280	0.94322	0.94583	0.94542	0.94640	0.94644	0.94637	0.94682	0.94612
$3/4$	25	0.95704	0.94960	0.95238	0.95394	0.95399	0.95435	0.95486	0.95602	0.95496	0.95413	0.95528	0.95512
	50	0.92331	0.92047	0.92689	0.93262	0.93294	0.93569	0.93749	0.93756	0.93903	0.93978	0.93865	0.94010
	100	0.90507	0.91674	0.93004	0.93514	0.93799	0.94186	0.94182	0.94339	0.94442	0.94371	0.94435	0.94539
	200	0.91349	0.92821	0.93894	0.94148	0.94485	0.94448	0.94545	0.94568	0.94641	0.94720	0.94659	0.94677
1	25	0.95838	0.94907	0.95133	0.95321	0.95329	0.95500	0.95481	0.95540	0.95501	0.95561	0.95653	0.95604
	50	0.92297	0.91848	0.92676	0.93297	0.93561	0.93571	0.93897	0.93789	0.93873	0.93963	0.93999	0.93860
	100	0.90439	0.91628	0.93168	0.93555	0.93911	0.94112	0.94388	0.94197	0.94374	0.94448	0.94376	0.94465
	200	0.91392	0.92911	0.93882	0.94231	0.94445	0.94526	0.94580	0.94720	0.94609	0.94678	0.94656	0.94689

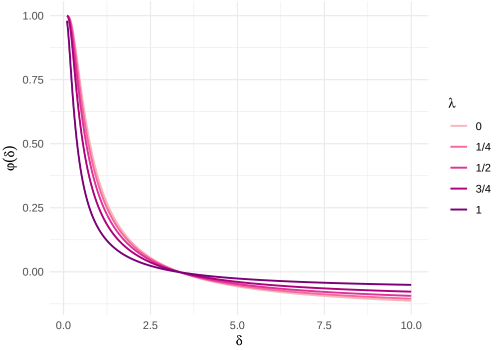
Case (ii) Estimated proportion of unbounded confidence intervals for $\alpha_1 = 0$ and $\alpha_2 = \alpha$

		δ											
λ	n	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
0	25	0.04323	0.34390	0.59523	0.72565	0.79500	0.83681	0.86009	0.87945	0.89125	0.89977	0.90575	0.91351
	50	0.00046	0.06814	0.27555	0.46459	0.59808	0.67919	0.73794	0.77697	0.80147	0.82258	0.83898	0.85155
	100	0.00000	0.00307	0.06709	0.21916	0.38353	0.50665	0.60092	0.66782	0.71704	0.75384	0.78261	0.80287
	200	0.00000	0.00000	0.00339	0.04296	0.14674	0.27505	0.38831	0.48683	0.56131	0.62163	0.67146	0.70826
$1/4$	25	0.04347	0.34649	0.59250	0.72672	0.79431	0.83809	0.85919	0.88009	0.89106	0.90042	0.90808	0.91133
	50	0.00055	0.06868	0.27334	0.46812	0.59563	0.68170	0.73660	0.77681	0.80518	0.82350	0.83972	0.85095
	100	0.00000	0.00276	0.06669	0.21894	0.38158	0.50909	0.59865	0.66812	0.71728	0.75305	0.77947	0.80199
	200	0.00000	0.00000	0.00312	0.04374	0.14696	0.27477	0.39136	0.48471	0.56279	0.62381	0.66961	0.71033
$1/2$	25	0.04465	0.34364	0.59437	0.72558	0.79642	0.83629	0.86221	0.87796	0.89231	0.90147	0.90778	0.91209
	50	0.00065	0.06685	0.27390	0.46826	0.59827	0.67999	0.73636	0.77567	0.80443	0.82317	0.84026	0.85367
	100	0.00000	0.00297	0.06555	0.22037	0.38292	0.50755	0.60225	0.66850	0.71689	0.75499	0.78170	0.80067
	200	0.00000	0.00000	0.00323	0.04437	0.14367	0.27125	0.39142	0.48852	0.56559	0.62480	0.67076	0.70790
$3/4$	25	0.04356	0.34629	0.59363	0.72549	0.79541	0.83684	0.86216	0.87998	0.89036	0.89868	0.90687	0.91310
	50	0.00062	0.06704	0.27315	0.46493	0.59400	0.68027	0.73482	0.77495	0.80353	0.82578	0.83904	0.85236
	100	0.00000	0.00281	0.06564	0.21857	0.38054	0.50763	0.60282	0.66944	0.71636	0.75261	0.78029	0.80250
	200	0.00000	0.00000	0.00332	0.04534	0.14651	0.26920	0.38818	0.48711	0.56099	0.62530	0.66985	0.70909
1	25	0.04496	0.34528	0.59287	0.72572	0.79495	0.83541	0.86192	0.87802	0.89087	0.90038	0.90962	0.91379
	50	0.00057	0.06728	0.27647	0.46779	0.59584	0.68066	0.73789	0.77533	0.80308	0.82349	0.84039	0.85193
	100	0.00000	0.00302	0.06616	0.22056	0.37842	0.51020	0.60304	0.66777	0.71746	0.75244	0.78099	0.80157
	200	0.00000	0.00000	0.00379	0.04656	0.14727	0.26943	0.39435	0.48524	0.56513	0.62289	0.66872	0.70737

Index

Conclusions

- The experiments carried out on the (shifted) Weibull distribution show that it succeeds at constructing confidence intervals with the desired coverage probability.
- Unfortunately, a large proportion of the obtained intervals turn out to be unbounded. Further investigation needs to be performed in this regard.



- The behaviour within other location-scale-skewness families is still to be explored.

Funding

Funding from grant PID2022-140585NB-I00 funded by MICIU/AEI/10.13039/501100011033 and "FEDER/UE".

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

Cofinanciado por la Unión Europea

AGENCIA ESTATAL DE INVESTIGACIÓN

Asymptotic confidence intervals for a skewness parameter by using OWA functions

Marina Iturrate–Bobes, Ignacio Montes and Raúl Pérez–Fernández

University of Oviedo

Universidad de Oviedo

21st International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems
Rome, Italy
June 15th–19th 2026