On the specific commutativity with Bayesian conditioning of systematic aggregation rules for probability measures

David Nieto-Barba

Sébastien Destercke

21^{st} International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems — IPMU'26

15-19 June 2026, Roma, Italia

1. Preliminaries
2. Introduction
3. Specific commutativity
   - Characterisation
   - The conditions are not necessary
   - A more involved example
4. Conclusions and future lines

1 Preliminaries

2 Introduction

3 Specific commutativity
  Characterisation
  The conditions are not necessary
  A more involved example

4 Conclusions and future lines

Beliefs, decision-making, conflict and aggregation.

Beliefs, decision-making, conflict and aggregation.

- Probability measures as a representation of **subjective beliefs** about the outcome of random phenomena.

Beliefs, decision-making, conflict and aggregation.

- Probability measures as a representation of **subjective beliefs** about the outcome of random phenomena.
- They entail **decision-making behaviour** from the expected utility theory perspective.

Beliefs, decision-making, conflict and aggregation.

- Probability measures as a representation of **subjective beliefs** about the outcome of random phenomena.
- They entail **decision-making behaviour** from the expected utility theory perspective.
- **Conflict** naturally arises when several opinions are put together.

Beliefs, decision-making, conflict and aggregation.

- Probability measures as a representation of **subjective beliefs** about the outcome of random phenomena.
- They entail **decision-making behaviour** from the expected utility theory perspective.
- **Conflict** naturally arises when several opinions are put together.
- **Aggregation rules** are methods assigning a single probability measure to a given set of m-probability measures.

Strong commutativity of aggregation with Bayes conditioning.

***Strong* commutativity of aggregation with Bayes conditioning.**

- For **any** conditioning event and set of individual beliefs:

  aggregating conditioned models $\equiv$ conditioning aggregated model **?**

Strong **commutativity of aggregation with Bayes conditioning.**

- For **any** conditioning event and set of individual beliefs:

  aggregating conditioned models $\equiv$ conditioning aggregated model **?**

- Otherwise, the group may be Dutch-bookable.

Strong **commutativity of aggregation with Bayes conditioning.**

- For **any** conditioning event and set of individual beliefs:

  aggregating conditioned models $\equiv$ conditioning aggregated model **?**

- Otherwise, the group may be Dutch-bookable.

Specific **commutativity of aggregation with Bayes conditioning.**

- And **given** a particular event and profile?

1. Preliminaries
2. Introduction
3. Specific commutativity
    - Characterisation
    - The conditions are not necessary
    - A more involved example
4. Conclusions and future lines

Let $\mathcal{X} := \{x_1, \ldots, x_n\}$ and $\mathbb{P}(\mathcal{X})$ the set of probability measures over $\mathcal{P}(\mathcal{X})$.

Let $\mathcal{X} := \{x_1, \ldots, x_n\}$ and $\mathbb{P}(\mathcal{X})$ the set of probability measures over $\mathcal{P}(\mathcal{X})$.

Definition (Aggregation rule and systematicity)

An **aggregation rule** *is a function* $\mathcal{A} : (\mathbb{P}(\mathcal{X}))^m \to \mathbb{P}(\mathcal{X})$*, where* $m \in \mathbb{N}$*.*

Let $\mathcal{X} := \{x_1, \ldots, x_n\}$ and $\mathbb{P}(\mathcal{X})$ the set of probability measures over $\mathcal{P}(\mathcal{X})$.

Definition (Aggregation rule and systematicity)

An **aggregation rule** *is a function* $\mathcal{A} : (\mathbb{P}(\mathcal{X}))^m \to \mathbb{P}(\mathcal{X})$, *where* $m \in \mathbb{N}$.
It is said to be **systematic** *whenever there exists* $F : [0,1]^m \to [0,1]$

Let $\mathcal{X} := \{x_1, \ldots, x_n\}$ and $\mathbb{P}(\mathcal{X})$ the set of probability measures over $\mathcal{P}(\mathcal{X})$.

Definition (Aggregation rule and systematicity)

An **aggregation rule** *is a function* $\mathcal{A} : (\mathbb{P}(\mathcal{X}))^m \to \mathbb{P}(\mathcal{X})$*, where* $m \in \mathbb{N}$*.*
It is said to be **systematic** *whenever there exists* $F : [0,1]^m \to [0,1]$ *such that for each* **profile** $(P_1, \ldots, P_m) \in (\mathbb{P}(\mathcal{X}))^m$*:*

$$\mathcal{A}(P_1, \ldots, P_m)(A) = F(P_1(A), \ldots, P_m(A)) \quad \forall A \subseteq \mathcal{X}.$$

Let $\mathcal{X} := \{x_1, \ldots, x_n\}$ and $\mathbb{P}(\mathcal{X})$ the set of probability measures over $\mathcal{P}(\mathcal{X})$.

Definition (Aggregation rule and systematicity)

An **aggregation rule** *is a function* $\mathcal{A} : (\mathbb{P}(\mathcal{X}))^m \to \mathbb{P}(\mathcal{X})$*, where* $m \in \mathbb{N}$*. It is said to be* **systematic** *whenever there exists* $F : [0,1]^m \to [0,1]$ *such that for each* **profile** $(P_1, \ldots, P_m) \in (\mathbb{P}(\mathcal{X}))^m$*:*

$$\mathcal{A}(P_1, \ldots, P_m)(A) = F(P_1(A), \ldots, P_m(A)) \quad \forall A \subseteq \mathcal{X}.$$

Definition (Linear averaging)

An aggregation rule $\mathcal{A}$ *is called a* **linear averaging** *if there exists* $\{\alpha_i\}_{i=1}^m \subseteq [0,1]$ *with* $\sum_{i=1}^m \alpha_i = 1$

Let $\mathcal{X} := \{x_1, \ldots, x_n\}$ and $\mathbb{P}(\mathcal{X})$ the set of probability measures over $\mathcal{P}(\mathcal{X})$.

Definition (Aggregation rule and systematicity)

An **aggregation rule** *is a function* $\mathcal{A} : (\mathbb{P}(\mathcal{X}))^m \to \mathbb{P}(\mathcal{X})$*, where* $m \in \mathbb{N}$*. It is said to be* **systematic** *whenever there exists* $F : [0,1]^m \to [0,1]$ *such that for each* **profile** $(P_1, \ldots, P_m) \in (\mathbb{P}(\mathcal{X}))^m$*:*

$$\mathcal{A}(P_1, \ldots, P_m)(A) = F(P_1(A), \ldots, P_m(A)) \quad \forall A \subseteq \mathcal{X}.$$

Definition (Linear averaging)

An aggregation rule $\mathcal{A}$ *is called a* **linear averaging** *if there exists* $\{\alpha_i\}_{i=1}^m \subseteq [0,1]$ *with* $\sum_{i=1}^m \alpha_i = 1$ *and* $\mathcal{A}(P_1, \ldots, P_m)(A) = \sum_{i=1}^m \alpha_i P_i(A)$*,* $\forall A$*.*

Let $\mathcal{X} := \{x_1, \ldots, x_n\}$ and $\mathbb{P}(\mathcal{X})$ the set of probability measures over $\mathcal{P}(\mathcal{X})$.

Definition (Aggregation rule and systematicity)

An **aggregation rule** *is a function* $\mathcal{A} : (\mathbb{P}(\mathcal{X}))^m \to \mathbb{P}(\mathcal{X})$*, where* $m \in \mathbb{N}$*. It is said to be* **systematic** *whenever there exists* $F : [0,1]^m \to [0,1]$ *such that for each* **profile** $(P_1, \ldots, P_m) \in (\mathbb{P}(\mathcal{X}))^m$*:*

$$\mathcal{A}(P_1, \ldots, P_m)(A) = F(P_1(A), \ldots, P_m(A)) \quad \forall A \subseteq \mathcal{X}.$$

Definition (Linear averaging)

An aggregation rule $\mathcal{A}$ *is called a* **linear averaging** *if there exists* $\{\alpha_i\}_{i=1}^m \subseteq [0,1]$ *with* $\sum_{i=1}^m \alpha_i = 1$ *and* $\mathcal{A}(P_1, \ldots, P_m)(A) = \sum_{i=1}^m \alpha_i P_i(A)$*,* $\forall A$*.*

Theorem (McConway, 1981)

An aggregation rule is systematic if and only if it is a linear average.

Definition (*Strong* commutativity)

For all $(P_1, \ldots, P_m) \in \mathbb{P}(\mathcal{X})$ *and* $B \subseteq \mathcal{X}$ *it holds:*

$$R(\cdot) := \mathcal{A}(P_1, \ldots, P_m)(\cdot|B) = \mathcal{A}(P_1(\cdot|B), \ldots, P_m(\cdot|B)).$$

Definition (*Strong* commutativity)

For all $(P_1, \ldots, P_m) \in \mathbb{P}(\mathcal{X})$ *and* $B \subseteq \mathcal{X}$ *it holds:*

$$R(\cdot) := \mathcal{A}(P_1, \ldots, P_m)(\cdot|B) = \mathcal{A}(P_1(\cdot|B), \ldots, P_m(\cdot|B)).$$

Equivalently, the following diagram commutes:

$$\begin{tikzcd}
\{P_i\}_{i=1}^{m} \arrow[r, "\mathcal{A}"] \arrow[d, "\cdot|_B"'] & \mathcal{A}(P_1, \ldots, P_m) \arrow[d, "\cdot|_B"'] \\
\{P_i(\cdot|B)\}_{i=1}^{m} \arrow[r, "\mathcal{A}"] & \mathsf{R}
\end{tikzcd}$$

Theorem (Russell et al., 2015)

$\left[\textit{systematic \& strong commutativity}\right]$

Theorem (Russell et al., 2015)

$$\left[\textit{systematic \& strong commutativity}\right] \Leftrightarrow \textit{dictatorship}$$

Theorem (Russell et al., 2015)

$$\big[\textit{systematic \& strong commutativity}\big] \Leftrightarrow \textit{dictatorship}$$

Definition (*Specific* commutativity)

Given $(P_1, \ldots, P_m) \in \mathbb{P}(\mathcal{X})$ *and* $B \subseteq \mathcal{X}$ *it holds:*

$$R(\cdot) := \mathcal{A}(P_1, \ldots, P_m)(\cdot|B) = \mathcal{A}(P_1(\cdot|B), \ldots, P_m(\cdot|B)).$$

Theorem (Russell et al., 2015)

$$\big[\textit{systematic \& strong commutativity}\big] \Leftrightarrow \textit{dictatorship}$$

Definition (*Specific* commutativity)

Given $(P_1, \ldots, P_m) \in \mathbb{P}(\mathcal{X})$ *and* $B \subseteq \mathcal{X}$ *it holds:*

$$R(\cdot) := \mathcal{A}(P_1, \ldots, P_m)(\cdot|B) = \mathcal{A}(P_1(\cdot|B), \ldots, P_m(\cdot|B)).$$

Proposition (Polina et al., 2025)

Let $(P_1, \ldots, P_m) \in (\mathbb{P}(\mathcal{X}))^m$, $B \subseteq \mathcal{X}$ *such that* $P_i(B) > 0$ *for each* i. *Then,*

$$\big[P_i(B) = P_j(B)\ \forall i, j\big] \Rightarrow \big[\textit{commutativity for any systematic rule}\big].$$

1 Preliminaries

2 Introduction

3 Specific commutativity
   Characterisation
   The conditions are not necessary
   A more involved example

4 Conclusions and future lines

Characterisation

Given $(P_1, \ldots, P_m) \in \mathbb{P}(\mathcal{X})$ and $B \subseteq \mathcal{X}$, for a systematic aggregation rule:

Given $(P_1, \ldots, P_m) \in \mathbb{P}(\mathcal{X})$ and $B \subseteq \mathcal{X}$, for a systematic aggregation rule:

$$
\begin{aligned}
&\mathcal{A}(P_1, \ldots, P_m)(A|B) = \mathcal{A}(P_1(\cdot|B), \ldots, P_m(\cdot|B))(A) \\
&\Leftrightarrow \frac{\sum_{i=1}^m \alpha_i P_i(A \cap B)}{\sum_{i=1}^m \alpha_i P_i(B)} = \sum_{i=1}^m \alpha_i P_i(A|B) \\
&\Leftrightarrow \sum_{i=1}^m \alpha_i P_i(A \cap B) = \left(\sum_{i=1}^m \alpha_i P_i(B)\right)\left(\sum_{i=1}^m \alpha_i P_i(A|B)\right) \Leftrightarrow \cdots \\
&\Leftrightarrow f(\alpha; A) := \sum_{i,j|i<j} \left[\alpha_i \alpha_j \big(P_i(B) - P_j(B)\big)\big(P_j(A|B) - P_i(A|B)\big)\right] = 0.
\end{aligned}
$$

Proposition

Specific commutativity holds for systematic rules iff:

$$f\big(\alpha;\{x\}\big) = 0 \quad \forall x \in \mathcal{X}.$$

Proposition

Specific commutativity holds for systematic rules iff:

$$f\big(\alpha;\{x\}\big) = 0 \quad \forall x \in \mathcal{X}.$$

In particular, it holds whenever either:

1. *The aggregation rule is dictatorial, i.e.* $\alpha_{i^*} = 1$ *for certain* i^**;*
2. *For each* i, j, $P_i(B) = P_j(B)$*;*

Proposition

Specific commutativity holds for systematic rules iff:

$$f\big(\alpha;\{x\}\big) = 0 \quad \forall x \in \mathcal{X}.$$

In particular, it holds whenever either:

1. *The aggregation rule is dictatorial, i.e.* $\alpha_{i^*} = 1$ *for certain* i^*;
2. *For each* i, j, $P_i(B) = P_j(B)$;
3. *For each* i, j, $P_i(\cdot|B) = P_j(\cdot|B)$;

Proposition

Specific commutativity holds for systematic rules iff:

$$f\big(\alpha; \{x\}\big) = 0 \quad \forall x \in \mathcal{X}.$$

In particular, it holds whenever either:

1. *The aggregation rule is dictatorial, i.e.* $\alpha_{i^*} = 1$ *for certain* i^**;*
2. *For each* i, j, $P_i(B) = P_j(B)$*;*
3. *For each* i, j, $P_i(\cdot|B) = P_j(\cdot|B)$*;*
4. *For every* i, j, *either:*

$$\alpha_i = 0, \quad \alpha_j = 0, \quad P_i(B) = P_j(B) \quad \textit{or} \quad P_i(\cdot|B) = P_j(\cdot|B).$$

The conditions are not necessary

Example

Let $n = m = 3$, $P_1 = (1/3, 2/3, 0)$, $P_2 = (2/3, 1/3, 0)$, $P_3 = (1/3, 1/3, 1/3)$ *and* $B = \{x_1, x_2\}$.

Example

Let $n = m = 3$, $P_1 = (1/3, 2/3, 0)$, $P_2 = (2/3, 1/3, 0)$, $P_3 = (1/3, 1/3, 1/3)$ *and* $B = \{x_1, x_2\}$. *Since* $P_1(B) = P_2(B) = 1$, *for* $A_1 = \{x_1\}$:

$$\begin{aligned} f(\alpha; A_1) &= \alpha_1\alpha_3\left(1 - \frac{2}{3}\right)\left(\frac{1}{2} - \frac{1}{3}\right) + \alpha_2\alpha_3\left(1 - \frac{2}{3}\right)\left(\frac{1}{2} - \frac{2}{3}\right) \\ &= \frac{\alpha_3}{18}(\alpha_1 - \alpha_2). \end{aligned}$$

Also, $f(\alpha; A_2) = -f(\alpha; A_1)$ *for* $A_2 = \{x_2\}$.

Example

Let $n = m = 3$, $P_1 = (1/3, 2/3, 0)$, $P_2 = (2/3, 1/3, 0)$, $P_3 = (1/3, 1/3, 1/3)$ *and* $B = \{x_1, x_2\}$. *Since* $P_1(B) = P_2(B) = 1$, *for* $A_1 = \{x_1\}$:

$$\begin{aligned} f(\alpha; A_1) &= \alpha_1\alpha_3\left(1 - \frac{2}{3}\right)\left(\frac{1}{2} - \frac{1}{3}\right) + \alpha_2\alpha_3\left(1 - \frac{2}{3}\right)\left(\frac{1}{2} - \frac{2}{3}\right) \\ &= \frac{\alpha_3}{18}(\alpha_1 - \alpha_2). \end{aligned}$$

Also, $f(\alpha; A_2) = -f(\alpha; A_1)$ *for* $A_2 = \{x_2\}$.

Hence, specific commutativity holds iff either $\alpha_3 = 0$ *or* $\alpha_1 = \alpha_2$.

Example

Let $n = m = 3$, $P_1 = (1/3, 2/3, 0)$, $P_2 = (2/3, 1/3, 0)$, $P_3 = (1/3, 1/3, 1/3)$ *and* $B = \{x_1, x_2\}$. *Since* $P_1(B) = P_2(B) = 1$, *for* $A_1 = \{x_1\}$:

$$\begin{aligned} f(\alpha; A_1) &= \alpha_1\alpha_3\left(1 - \frac{2}{3}\right)\left(\frac{1}{2} - \frac{1}{3}\right) + \alpha_2\alpha_3\left(1 - \frac{2}{3}\right)\left(\frac{1}{2} - \frac{2}{3}\right) \\ &= \frac{\alpha_3}{18}(\alpha_1 - \alpha_2). \end{aligned}$$

Also, $f(\alpha; A_2) = -f(\alpha; A_1)$ *for* $A_2 = \{x_2\}$.

Hence, specific commutativity holds iff either $\alpha_3 = 0$ *or* $\alpha_1 = \alpha_2$*.This includes non-dictatorial weights:*

$$\{(t, t, 1 - 2t) \mid t \in (0, 0.5]\}.$$

A more involved example

Example

Consider $n = 5$ *intended threat levels: "invasion"* (x_1), *"siege"* (x_2), *"military hostilities in the border"* (x_3), *"verbal threats"* (x_4) *or "absence of any threat or aggression"* (x_5). *The event* $B = \{x_1, x_2, x_3\}$ *embraces the possibilities with different degrees of military hostilities.*

A more involved example

Example

Consider $n = 5$ *intended threat levels: "invasion"* (x_1)*, "siege"* (x_2)*, "military hostilities in the border"* (x_3)*, "verbal threats"* (x_4) *or "absence of any threat or aggression"* (x_5)*. The event* $B = \{x_1, x_2, x_3\}$ *embraces the possibilities with different degrees of military hostilities.*

Consider $m = 3$ *federations,* $P_1 = (0.3, 0.3, 0.3, 0.07, 0.03)$*,* $P_2 = (0.05, 0.2, 0.05, 0.35, 0.35)$*,* $P_3 = (0.25, 0.1, 0.25, 0.2, 0.2)$*.*

A more involved example

Example

Consider $n = 5$ *intended threat levels: "invasion"* (x_1)*, "siege"* (x_2)*, "military hostilities in the border"* (x_3)*, "verbal threats"* (x_4) *or "absence of any threat or aggression"* (x_5)*. The event* $B = \{x_1, x_2, x_3\}$ *embraces the possibilities with different degrees of military hostilities.*

Consider $m = 3$ *federations,* $P_1 = (0.3, 0.3, 0.3, 0.07, 0.03)$*,* $P_2 = (0.05, 0.2, 0.05, 0.35, 0.35)$*,* $P_3 = (0.25, 0.1, 0.25, 0.2, 0.2)$*. Thus:*

$$f(\alpha;\{x_1\}) = \frac{0.3}{12}\big(-0.4\alpha_1\alpha_2 + \alpha_1\alpha_3 - 3\alpha_2\alpha_3\big) = 0$$

Example

Consider $n = 5$ *intended threat levels: "invasion"* (x_1)*, "siege"* (x_2)*, "military hostilities in the border"* (x_3)*, "verbal threats"* (x_4) *or "absence of any threat or aggression"* (x_5)*. The event* $B = \{x_1, x_2, x_3\}$ *embraces the possibilities with different degrees of military hostilities.*

Consider $m = 3$ *federations,* $P_1 = (0.3, 0.3, 0.3, 0.07, 0.03)$*,* $P_2 = (0.05, 0.2, 0.05, 0.35, 0.35)$*,* $P_3 = (0.25, 0.1, 0.25, 0.2, 0.2)$*. Thus:*

$$f(\alpha; \{x_1\}) = \frac{0.3}{12}\big(-0.4\alpha_1\alpha_2 + \alpha_1\alpha_3 - 3\alpha_2\alpha_3\big) = 0$$

and $2f(\alpha; \{x_1\}) = -f(\alpha; \{x_2\}) = 2f(\alpha; \{x_3\})$*.*

A more involved example

Example

Consider $n = 5$ *intended threat levels: "invasion"* (x_1)*, "siege"* (x_2)*, "military hostilities in the border"* (x_3)*, "verbal threats"* (x_4) *or "absence of any threat or aggression"* (x_5)*. The event* $B = \{x_1, x_2, x_3\}$ *embraces the possibilities with different degrees of military hostilities.*

Consider $m = 3$ *federations,* $P_1 = (0.3, 0.3, 0.3, 0.07, 0.03)$*,* $P_2 = (0.05, 0.2, 0.05, 0.35, 0.35)$*,* $P_3 = (0.25, 0.1, 0.25, 0.2, 0.2)$*. Thus:*

$$f(\alpha; \{x_1\}) = \frac{0.3}{12}\big(- 0.4\alpha_1\alpha_2 + \alpha_1\alpha_3 - 3\alpha_2\alpha_3\big) = 0$$

and $2f(\alpha; \{x_1\}) = -f(\alpha; \{x_2\}) = 2f(\alpha; \{x_3\})$*. It is easily checked that the desired equalities are satisfied, for instance, for the following vector of weights:* $\alpha = (\alpha_1, \alpha_2, \alpha_3) = ({}^5/_7, {}^1/_7, {}^1/_7)$*.*

A more involved example

Example

On the other hand, slightly modifying the last example: changing P_3 *by*

$$P_3 = (0.15, 0.3, 0.15, 0.1, 0.2),$$

yields that $f(\alpha; \{x_1\})$ *is proportional to* $-0.4\alpha_1\alpha_2 - \alpha_1\alpha_3 - \alpha_2\alpha_3$

A more involved example

Example

On the other hand, slightly modifying the last example: changing P_3 *by*

$$P_3 = (0.15, 0.3, 0.15, 0.1, 0.2),$$

yields that $f(\alpha; \{x_1\})$ *is proportional to* $-0.4\alpha_1\alpha_2 - \alpha_1\alpha_3 - \alpha_2\alpha_3$, *hence dictatorships would be the only solutions.*

1 Preliminaries

2 Introduction

3 Specific commutativity
Characterisation
The conditions are not necessary
A more involved example

4 Conclusions and future lines

Conclusions:

- a system of quadratic equations characterises the weights for which specific commutativity holds;

Conclusions:

- a system of quadratic equations characterises the weights for which specific commutativity holds;
- allows to identify non-dictatorial systematic aggregation rules that commute with conditioning;
- several sufficent conditions were derived (and shown to not be necessary).

Conclusions:

- a system of quadratic equations characterises the weights for which specific commutativity holds;
- allows to identify non-dictatorial systematic aggregation rules that commute with conditioning;
- several sufficent conditions were derived (and shown to not be necessary).

Future lines:

- How to integrate this results within the framework of dynamic rationality;

Conclusions:

- a system of quadratic equations characterises the weights for which specific commutativity holds;
- allows to identify non-dictatorial systematic aggregation rules that commute with conditioning;
- several sufficent conditions were derived (and shown to not be necessary).

Future lines:

- How to integrate this results within the framework of dynamic rationality;
- its relation with external Bayesianism;

Conclusions:

- a system of quadratic equations characterises the weights for which specific commutativity holds;
- allows to identify non-dictatorial systematic aggregation rules that commute with conditioning;
- several sufficent conditions were derived (and shown to not be necessary).

Future lines:

- How to integrate this results within the framework of dynamic rationality;
- its relation with external Bayesianism;
- thorough analysis of its relation to Dutch-books;

Conclusions:

- a system of quadratic equations characterises the weights for which specific commutativity holds;
- allows to identify non-dictatorial systematic aggregation rules that commute with conditioning;
- several sufficent conditions were derived (and shown to not be necessary).

Future lines:

- How to integrate this results within the framework of dynamic rationality;
- its relation with external Bayesianism;
- thorough analysis of its relation to Dutch-books;
- extension to imprecise aggregation rules.

References

de Finetti, B.: Teoria delle Probabilità. Einaudi, Turin (1970)

Genest, C.: A characterization theorem for externally bayesian groups. The Annals of Statistics 12(3), 1100–1105 (1984)

Gordienko, P., Jansen, C., Augustin, T., Rechenauer, M.: Consensus in motion: A case of dynamic rationality of sequential learning in probability aggregation. In: Sauerwald, K., Thimm, M. (eds.) Proceedings of ECSQARU'25, vol. LNAI 16099, pp. 178–190. Springer Nature (2026)

McConway, K.J.: Marginalization and linear opinion pools. Journal of the American Statistical Association 76(374), 410–414 (1981)

Russell, J., Hawthorne, J., Buchak, L.: Groupthink. Philosofical Studies 172, 1287–1309 (2015)

Savage, L.J.: The Foundations of Statistics. Dover, New York (1972), second revised edition, first published 1954.

Stewart, R.T., Quintana, I.O.: Learning and pooling, pooling and learning. Erkenntis 83, 369–3895 (2018)

Stone, M.: The opinion pool. The Annals of Mathematical Statistics, 32(4), 1339–1342 (1961)

Von Neumann, J., Morgenstern, O.: Theory of Games and Economic Behaviour. Princeton University Press, Princeton (1972), third revised edition, first published 1944.

Wagner, C.: Allocation, Lehrer models, and the consensus of probabilities. Theory and Decision 14, 207–220 (1982)

Funding

Proyect PID2022-140585NB-I00 funded by MICIU/AEI/10.13039/501100011033 y FEDER, UE

Predoctoral program Severo Ochoa NAC-AT-PUB-ASV-2025 BP24-18 funded by the Principality of Asturias

On the specific commutativity with Bayesian conditioning of systematic aggregation rules for probability measures

David Nieto-Barba

Sébastien Destercke

21st International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems — IPMU'26

15-19 June 2026, Roma, Italia